Characterizations of Narrowband MIMO Channels

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***Abstract*—Computational electromagnetics are useful for examining communication link performance in complex propagation environments. In this paper we illustrate how different characterizations of narrowband multiple-input multiple-output (MIMO) channels, including the mutual information, received power, and effective rank, behave as a function of location. Large-scale simulation results are presented for a 7x7 MIMO system operating in an outdoor urban area. We find that the differences between MIMO channels are more visible when the effects of path loss are removed from the calculations.**

I. Introduction

MIMO (multiple-input multiple-output) communication is one of the most promising technologies for achieving high data rates with limited bandwidth [1]. Deployment of real MIMO systems, however, will require characterizing the channel in terms of realistic electromagnetics. In this paper we study the problem of characterizing a narrowband MIMO communication channel using site-specific propagation modelling [2]. To provide context, consider the following equivalent discrete-time model of a MIMO system (assuming perfect timing and synchronization)

$$\mathbf{y}(n) \;=\; \sqrt{\frac{E_s}{M_t}}\mathbf{H}\mathbf{s}(n) \;+\; \mathbf{v}(n) \tag{1}$$

where $\{\mathbf{s}(n)\}_{n=0}^{N-1}$ is a sequence of transmitted vectors, $\mathbf{H}$ is the $M_r \times M_t$ matrix channel assumed constant over the N symbol periods (known perfectly to the receiver but not the transmitter), $\mathbf{v}(n)$ is a realization of a circular complex Gaussian distribution with zero mean and variance $N_o\mathbf{I}_{M_r}$, and E_s corresponds to the transmit energy assuming that $\mathcal{E}_{\mathbf{s}_n}\|\mathbf{s}_n\| = 1$ for $n = 1, 2, \ldots, M_t$. The channel characterization problem is to find a simple function of a set of channel realizations $\mathcal{H} := \{\mathbf{H}^{(n)}\}_{n=0}^{N-1}$ that indicates channel quality according to some performance metric. Allowing $N > 1$ gives the possibility of using multiple snapshots obtained in a local area to isolate and average out the small-scale fading effects of the channel.

In the scalar case ($M_r = M_t = 1$), it is known that "good" channels have large amplitude. The MIMO case is more uncertain since it depends on the choice of space-time code, receiver algorithm (ML receiver, MMSE, etc), and the performance metric (error probability or asymptotic criteria) [3] [4].

II. MIMO Channel Characterization

Previous work has used only the mutual information in (3) to define good channels (e.g., [5]). There are however a variety of candidate characterization functions for narrowband MIMO systems each fulfilling a different set of requirements. Possible functions include the average signal strength

$$P(\mathcal{H}) \;=\; \frac{1}{N}\sum_{n=0}^{N-1}\|\mathbf{H}^{(n)}\|_F^2, \tag{2}$$

the average mutual information

$$\bar{I}(\mathcal{H}) \;=\; \frac{1}{N}\sum_{n=0}^{N-1}\log\det\left(\mathbf{I}_{M_r} \;+\; \frac{E_s}{N_o M_t}\mathbf{H}^{(n)}\mathbf{H}^{(n)H}\right), \tag{3}$$

the normalized average mutual information

$$\bar{I}_s(\mathcal{H}) \;=\; \frac{1}{N}\sum_{n=0}^{N-1}\log\det\left(\mathbf{I}_{M_r} \;+\; \frac{E_s}{N_o}\frac{1}{\alpha}\mathbf{H}^{(n)}\mathbf{H}^{(n)H}\right) \tag{4}$$

where $\alpha = \frac{1}{N M_t M_r}\sum_{n=0}^{N-1}\|\mathbf{H}^{(n)}\|_F^2$ is an estimate of the path-loss, and the "effective rank," (e.g. the effective number of non-negligible singular values)

$$\bar{R}(\mathcal{H}) \;=\; \frac{1}{N}\sum_{n=0}^{N-1}R(\mathbf{H}^{(n)}). \tag{5}$$

$R\{\mathbf{H}^{(n)}\} := \sum_m I\left(\lambda_m > K\lambda_{max}\right)$ is the number of singular values of $\mathbf{H}^{(n)}$ greater than a pre-defined fraction K of the maximum singular value. The signal strength in (2) is the natural extension of its scalar counterpart. The average mutual information gives an estimate of the throughput that can be achieved with no knowledge of the channel and a Gaussian transmitted signal with identity covariance. The difference between (3) and (4) is that (4) removes the effect of the average signal strength (i.e. assuming perfect transmit power control). The effective rank is a generalized estimate of the number of "spatial data pipes" that are available in the matrix channel and, unlike (3) and (4), does not depend on SNR.

III. Simulation Results

To illustrate the behavior of different channel characterizations as a function of location, we used the FASANT electromagnetic ray tracing (ERT) software [7] [8] to simulate channels in downtown Austin. A base station with a 7-element circular array and half-wavelength spacing (carrier of $1.9GHz$) was placed on a $20m$ tower in the center of the simulation model. In every local city street area of $5m^2$ we obtained a set of $N = 9$ channel measurements $\mathcal{H} := \{\mathbf{H}^{(n)}\}_{n=0}^{N-1}$ for a mobile unit with an identical array $1.5m$ above the ground. To compute the noise power we assumed a bandwidth of $1MHz$ (though the channel is modelled as narrow-band).

In Fig. 1(a) we plot an indication of the line-of-sight (LOS) regions. Regions that are non-line-of-sight (NLOS) receive signals on through reflection or diffraction. Comparing with the average signal strength in Fig. 1(a) and Fig. 1(b), there is an expected strong correlation between LOS and high signal strength. From Fig. 1(c), regions with significant signal strength also support a large $\bar{I}(\mathcal{H})$ since this corresponds to high SNR. In real systems, however, the effective SNR is often limited due to distortion in the RF components of the radio thus Fig. 1(c) may be optimistic. To separate the effect of path loss, assume the use of transmit power control that removes large-scale differences in power. The result, $\bar{I}_s(\mathcal{H})$, is illustrated in Fig. 1(d). Comparing with Fig. 1(c), the normalized mutual information is much less dependent on high signal strength and is more strongly a function of the surroundings in each location. In Fig. 1(e) we plot $\bar{R}(\mathcal{H})$ which is an estimate of the effective rank of the channel. Comparing Fig. 1(c) and Figs. 1(d)-(e) we see that without normalization the mutual information may be high even when the effective rank, and thus number of "spatial data pipes," is small.

To conclude, in MIMO systems, comparisons of the mutual information (or equivalently the capacity) need to be qualified when including large-scale effects in the channel. Normalized measures, such as $\bar{I}_s(\mathcal{H})$ or $\bar{R}(\mathcal{H})$, are insensitive to path loss and thus convey better intuition about the number of "spatial data pipes" in the channel. The best channel characterization, however, is still open to question.

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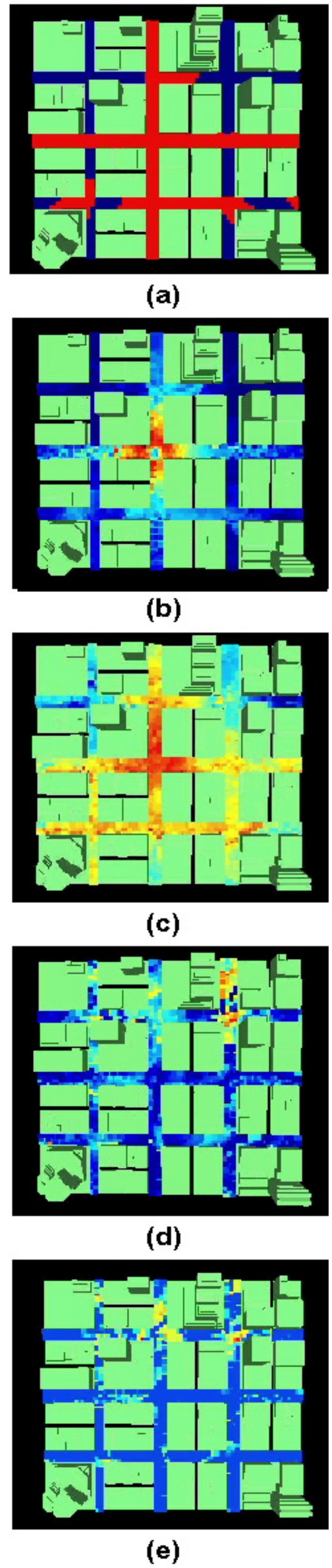

Fig. 1. (a) Indication of line-of-sight regions (b) Average received power (c) Average mutual information (d) Normalized average mutual information (e) Effective rank